SCIENCE FOR GLASS PRODUCTION

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MATHEMATICAL MODEL FOR CALCULATING THE TEMPERATURE FIELD AND STRESSES IN GLASS TUBES UPON ANNEALING

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A mathematical model which describes the distribution of the temperature field and stresses originated upon cooling of a glass tube is proposed. The model is developed proceeding from the solution of a conjugated heat transfer problem. The model is presented in the form of a system of finite-difference equations and makes it possible to analyze the dynamics of the annealing process in various cooling regimes using computer simulation.

The process of annealing that determines the character of distribution of temporal and residual stresses in a glass article plays an important role in glass tube production. Unsatisfactory distribution of residual stresses reduces the service characteristics of the glass tubes and results in their rejection. Exceeding the rated thermoelastic strains can lead to spontaneous fracture of the article in the stage of annealing. A mathematical model should be formulated to substantiate the choice of the annealing regime. The models developed earlier provide for determination of the temperature field and stresses in the wall of the glass tube (or any other article of the same shape) upon static cooling [1 – 3] they did not allow describing the dynamic aspects of the heat transfer processes and initiation of stresses.

Thus, the goal is to develop a mathematical model which describes the heat transfer processes, initiation and distribution of stresses in a glass tube wall upon annealing. The model is also adaptable to optimization of annealing of glass tubes and other cylindrical glasswork, e.g., bottles.

Consider a glass tube of radii R_1 and R_2 (Fig. 1). Let the temperature gradient be across the tube wall alone. Thus the temperature field obeys the equation

$$\frac{\partial T_1(r,t)}{\partial t} = \frac{\lambda_1}{c_1 \rho_1}\left(\frac{\partial^2 T_1(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial T_1(r,t)}{\partial r}\right), \quad (1)$$

where $T_1(r, t)$ is the temperature of the glass layer; r is the layer radius ($R_2 \le r \le R_1$); t is the current time; λ_1 is the heat conduction; c_1 and ρ_1 are the heat capacity and density of the glass, respectively.

The initial conditions are:

$$T_1(r, t_0) = T_0,$$

where t_0 and T_0 are the initial time and initial temperature of the glass, respectively.

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The boundary conditions outside the tube obey the equation

$$\lambda_1 \frac{\partial T_1(r,t)}{\partial t} = h_1 \left(T_1(r,t) - T_a(t)\right), \quad (2)$$

where $r = R_1$ is the external diameter of the tube; h_1 is the heat transfer coefficient on the exterior surface of the tube; and $T_a(t)$ is the ambient temperature.

The boundary conditions inside the tube are given by the following system of equations:

$$\begin{cases} \lambda_1 \dfrac{\partial T_1(r,t)}{\partial t} = \lambda_2 \dfrac{\partial T_2(r,t)}{\partial t}, \\ T_1(r,t) = T_2(r,t) \end{cases} \quad (3)$$

where $r = R_2$ is the internal diameter of the tube; λ_2 and $T_2(r, t)$ are the heat conduction and air temperature inside the tube, respectively.

The analysis of boundary conditions (3) leads to the necessity of calculating the temperature field of the air inside the tube which obeys the equation:

$$\frac{\partial T_2(r,t)}{\partial t} = \frac{\lambda_2}{c_2 \rho_2}\left(\frac{\partial^2 T_2(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial T_2(r,t)}{\partial r}\right), \quad (4)$$

where $0 < r \le R_2$; c_2 and ρ_2 are the heat capacity and density of the air.

The initial conditions are given by the equation:

$$T_2(r, t_0) = T_0.$$

The presence of complicated conjugating boundary conditions (3), uncommon to heat transfer theory, is attributed to the fact that air-glass heat transfer in the closed space of the tube proceeds mainly through heat transmission rather than via convection because the temperature is constant throughout the inner surface of the tube (with no regard for heat transmission at the ends of the tube).

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Let us divide the inner air space and the tube wall into N layers with spacing of

$$\Delta r_2 = \frac{R_2}{N} \text{ and } \Delta r_1 = \frac{R_1 - R_2}{N}.$$

within the range of $(0, R_2)$ and (R_2, R_1) (see Fig. 1.).

Using network method [4] with due regard for boundary conditions (2), (3) we obtain from Eqs. (1), (4) a system of finite-difference equations for calculation of the temperature field of the glass tube:

$$\begin{cases}
\dfrac{T_1[i,k+1]-T_1[i,k\,]}{\Delta t}=\dfrac{\lambda_1\,(T_1[i,k\,])}{c_1\,\rho_1}\left(\dfrac{T_1[i+1,k\,]-2T_1[i,k\,]+T_1[i-1,k\,]}{\Delta r_1^{\,2}}+\dfrac{1}{r_1[i\,]}\dfrac{T_1[i+1,k\,]-T_1[i,k\,]}{\Delta r_1}\right), & i=2\ldots N-1;\\
\dfrac{T_1[i,k+1]-T_1[i,k\,]}{\Delta t}=\dfrac{\lambda_1\,(T_1[i,k\,])}{c_1\,\rho_1}\left(\dfrac{T_1[i+1,k\,]-T_1[i,k\,]}{\Delta r_1^{\,2}}+\left(\dfrac{1}{\Delta r_1}+\dfrac{1}{r_1[i\,]}\right)\dfrac{h_1\,(T_1[i,k\,]-T_a\,[k\,])}{\lambda_1\,\Delta r_1}\right), & i=1;\\
\dfrac{T_1[i,k+1]-T_1[i,k\,]}{\Delta t}=\dfrac{\lambda_1\,(T_1[i,k\,])}{c_1\,\rho_1}\left(\dfrac{T_1[i-1,k\,]-T_1[i,k\,]}{\Delta r_1^{\,2}}+\left(\dfrac{1}{\Delta r_1}+\dfrac{1}{r_1[i\,]}\right)\dfrac{\lambda_2\,(T_2[j+1,k\,]-T_2[j,k\,])}{\lambda_1\,\Delta r_2}\right), & i=N, j=1;\\
\dfrac{T_2[j,k+1]-T_2[j,k\,]}{\Delta t}=\dfrac{\lambda_2}{c_2\,\rho_2}\left(\dfrac{T_2[j+1,k\,]-2T_2[j,k\,]+T_2[j-1,k\,]}{\Delta r_2^{\,2}}+\dfrac{1}{r_2[j\,]}\dfrac{T_2[j+1,k\,]-T_2[j,k\,]}{\Delta r_2}\right), & j=2\ldots N-1;\\
\dfrac{T_2[j,k+1]-T_2[j,k\,]}{\Delta t}=\dfrac{\lambda_2}{c_2\,\rho_2}\left(\dfrac{T_2[j-1,k\,]-T_2[j,k\,]}{\Delta r_2^{\,2}}\right), & j=N;\\
T_2[j,k+1]=T_1[i,k+1], \quad j=1, i=N,
\end{cases} \tag{5}$$

where i and j are the sequence numbers of the layer (i, $j = 1 \ldots N$) (relevant to glass and air, respectively); k is the calculation step; $T_1\,[i, k]$ is the temperature of the i-th glass layer at step k; $\lambda_1\,(T_1\,[i, k\,])$ is the effective heat conduction of the glass

$$\lambda_1\,(T_1\,[i, k]) = 1.08 + 4.19 \times 10^{-4} + 1.82 \times 10^{-6}(R_1 - R_2)^{0.73}\,(T_1\,[i, k] - 273)^{2.5},$$

calculated with allowance for the radiation component of heat transfer; $T_2\,[j, k]$ is the temperature of the j-th air layer at step k; $r_1\,[i] = (R_1 - \Delta r_1\, i)$ is the radius of the i-th glass layer; and $r_2\,[j] = (R_2 - \Delta r_2 j)$ is the radius of the j-th layer of air.

The calculation step (in time) should be determined proceeding from the convergence of the solution of system (5):

$$\Delta t \;\alpha\; \frac{c_2\,\rho_2\,\Delta r_2^{\,2}}{3\lambda_2}.$$

An algorithm for calculating the temporal and residual stresses in the glass tube wall was developed on the basis of the data obtained in [5 – 7]. The same problem was solved there for a glass plate using the relaxation theory of glass melting. The algorithm developed is represented by a system of finite-difference equations:

$$\begin{cases}
T_1^{\,f}\,[i,k+1]=T_1[i,k\,]+\left(T_1^{\,f}\,[i,k\,]-T_1[i,k\,]\right)\exp\left(-\left(\dfrac{\Delta t}{\tau_1\left(T_1[i,k\,],T_1^{\,f}\,[i,k\,]\right)}\right)^{b_1}\right);\\
r_d\,[i,k+1]=r_0\,[i\,]\left(1+\alpha_1\left(T_1^{\,f}\,[i,k\,]-T_0\right)+\alpha_2\left(T_1[i,k\,]-T_1^{\,f}\,[i,k\,]\right)\right);\\
r_f\,[i,k+1]=r_d\,[i,k+1]-\sigma_{re}\,[i,k\,]\dfrac{1-\mu}{E};\\
\Delta r_f\,[i,k+1]=r_f\,[i,k+1]-r_f\,[i,k\,];\\
\Delta r\,[k+1]=\left(\displaystyle\sum_{i=1}^{N}\Delta r_f\,[i,k+1]\right)\dfrac{1}{N};\\
\Delta\sigma[i,k+1]=\dfrac{E}{1-\mu}\left(\left(r_d\,[i,k+1]-\Delta r\,[k+1]\right)-r_d\,[i,k\,]\right);\\
\sigma[i,k+1]=\left(\sigma[i,k\,]+\Delta\sigma[i,k+1]\right)\exp\left(-\left(\dfrac{\Delta t}{\tau_2\left(T_1[i,k\,],T_1^{\,f}\,[i,k\,]\right)}\right)^{b_2}\right);\\
\sigma_{re}\,[i,k+1]=\sigma_{re}\,[i,k\,]+\sigma[i,k+1]\left(1-\exp\left(-\left(\dfrac{\Delta t}{\tau_2\left(T_1[i,k\,],T_1^{\,f}\,[i,k\,]\right)}\right)^{b_2}\right)\right),
\end{cases} \tag{6}$$

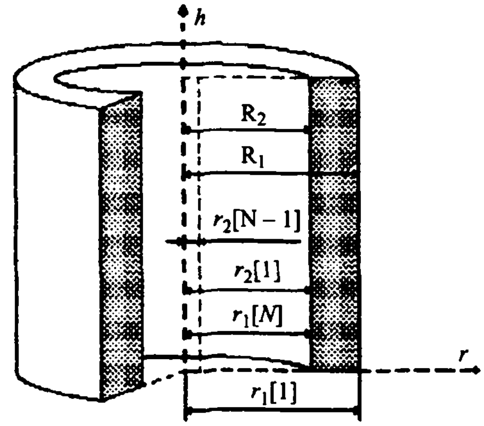

Fig. 1. Sectioning of a glass tube.

where $T_1^f\,[i,k]$ is the fictitious temperature of the i-th glass layer at step k; $\tau_1\left(T_1[i,k\,],T_1^f\,[i,k\,]\right)$ is the time of fictitious temperature relaxation

$$\tau_1\left(T_1[i,k\,],T_1^f\,[i,k\,]\right)=\frac{\eta\left(T_1[i,k\,],T_1^f\,[i,k\,]\right)}{K_1};$$

$\eta\left(T_1[i,k\,],T_1^f\,[i,k\,]\right)$ is the viscosity of the glass determined from the set of equations:

$$\eta\left(T_1[i,k\,],T_1^f\,[i,k\,]\right)= =A+\frac{B_1}{T_1^f\,[i,k\,]-C}+B_2\left(\frac{1}{T_1[i,k\,]}-\frac{1}{T_1^f\,[i,k\,]}\right),$$

where A, B_1, B_2, C, K_1 are constant for each glass composition; b_1 is the index of fictitious temperature relaxation and depends on the glass composition; $r_d\,[i,k]$ is the dilatometric radius of the i-th glass layer at step k; $r_0\,[i]$ is the radius of the i-th glass layer at the beginning of the calculation; α_1 and α_2 are the temperature expansion coefficients of the glass in the plastic and glassy state, respectively; $r_f[i,k]$ is the free radius of the i-th glass layer at step k; $\Delta r_f[i,k]$ is the change in the free radius of the i-th glass layer at step k; $\Delta r\,[k]$ is the actual change in tube radius at step k; $\sigma_{re}\,[i,k]$ is the sum of stresses relaxed in the i-th glass layer at step k; $\Delta\sigma[i,k]$ are the stresses originated in the i-th glass layer at step k; E is Young's modulus; μ is Poisson's coefficient; $\sigma[i,k]$ is the current value of the stress in the i-th glass layer at step k; $\tau_2\left(T_1[i,k\,],T_1^f\,[i,k\,]\right)$ is the stress relaxation time determined by

$$\tau_2\left(T_1[i,k\,],T_1^f\,[i,k\,]\right)=\frac{\eta\left(T_1[i,k\,],T_1^f\,[i,k\,]\right)}{K_2},$$

where K_2 is the constant which depends on the glass composition; and b_2 is the index of stress relaxation which also depends on the glass composition.

Figure 2 illustrates the process of glass tube annealing calculated with the following parameters of relaxation models (5), (6): $N=10$, $R_1=0.26$ m, $R_2=0.25$ m, $c_1=1200$ J/(kg · K), $\rho_1=2500$ kg/m^3, $c_2=1000$ J/(kg · K), $\rho_2=0.5$ kg/m^3, $\lambda_2=0.06$ W/(m · K), $E=7\cdot10^{10}$ Pa, $\mu=0.22$, $\alpha_1=3.3\cdot10^{-5}\,\mathrm{K}^{-1}$, $\alpha_2=1.1\cdot10^{-5}\,\mathrm{K}^{-1}$, $b_1=0.65$, $b_2=0.55$, $K_1=10^{9.8}$,

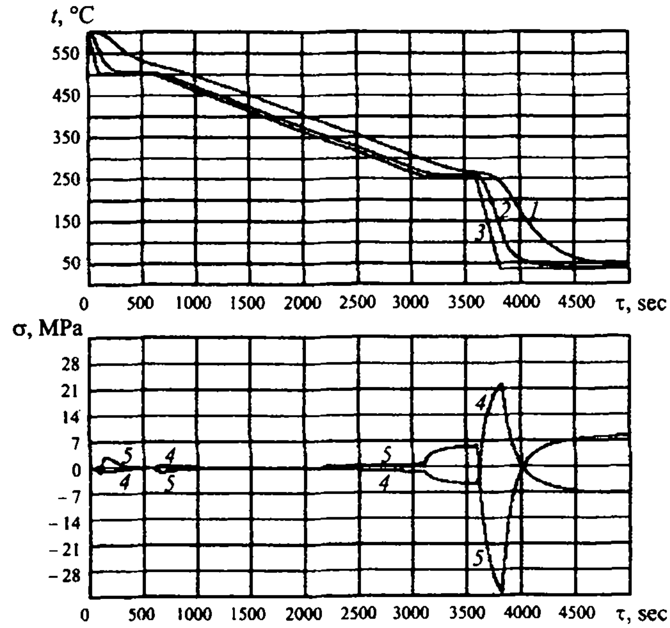

Fig. 2. Simulation of the glass tube annealing process: *1*) air temperature in the center of the tube (along the cylinder axis); *2*) temperature of the inner surface of the glass tube; *3*) air temperature outside the tube (ambient temperature); *4*) stresses in the external tube surface; *5*) stresses in the internal tube surface.

$A=-2.78$, $B_1=5574$ K, $B_2=15{,}500$ K, $C=474$ K, $T_0=873$ K, $h_1=50$ W/(m^2 · K).

The solution of the problem considered makes it possible to develop a mathematical model for describing the process of stress initiation and distribution of temperatures and stresses upon cooling of a glass tube. The model proposed makes it possible to analyze the dynamics of annealing in various cooling regimes using computer simulation, thus providing for an evaluation of the efficiency of the annealing regime. The model can be used to analyze the dynamics of annealing in different cooling regimes based on computer simulation, making it possible to assess the efficiency of an existing annealing regime or comparing and then selecting the most efficient regime (process control). The model developed can also give grounds for optimal process control, i.e., for optimization of cooling on the basis of selected criterion of quality within prescribed limits.

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